

Nucleon Structure Functions in a Chiral Soliton Model*

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The computation of nucleon structure functions within the Nambu–Jona–Lasinio chiral soliton model is outlined. After some technical remarks on the issue of regularization numerical results for the both unpolarized and polarized structure functions are presented. The generalization to flavor SU(3) is sketched.

1. THE CHIRAL SOLITON MODEL

The bosonized form of the Nambu–Jona–Lasinio (NJL) action [1]

$$\mathcal{A}[S, P] = -iN_C \text{Tr}_\Lambda \log [i\rlap{\not{D}} - (S + i\gamma_5 P)] - \frac{1}{4G} \int d^4x \text{tr} [S^2 + P^2 + 2\hat{m}_0(S + iP)] , \quad (1)$$

represents the starting point for considering nucleon structure functions in a chiral soliton model. The action \mathcal{A} is a functional of respectively scalar and pseudoscalar fields S and P which are matrices in flavor space. The VEV, $\langle S \rangle = m$ is obtained from the gap-equation and measures the dynamical breaking of chiral symmetry. For apparent reasons m is called the constituent quark mass (matrix). The regularization of the quadratically divergent quark loop is indicated by the cut-off Λ . Its value as well as the coupling constant G and the current quark mass \hat{m}_0 are adjusted to the phenomenological meson parameters m_π and f_π , leaving only a single free parameter, commonly chosen to be m . The static soliton is constructed from the hedgehog *ansatz* on the chiral circle

$$S + i\gamma_5 P = m \exp(i\vec{\tau} \cdot \hat{r} \gamma_5 \Theta(r)) =: m U_5 . \quad (2)$$

This defines a Dirac Hamiltonian $h = \vec{\alpha} \cdot \vec{p} + m\beta U_5$ with eigenvalues ϵ_α . The latter parameterize the regularized energy functional extracted from the action (1)

$$E[\Theta] = N_C \left(\epsilon_{\text{val}} - \frac{1}{2} \sum_\alpha \frac{1}{\Lambda} |\epsilon_\alpha| \right) . \quad (3)$$

Here the subscript “val” refers to the distinct valence quark level which is strongly bound in the background of the hedgehog. The soliton is finally constructed by extremizing the

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functional (3). Details of this approach and the computation of static nucleon properties are extensively discussed in the literature [2].

2. REMARKS ON REGULARIZATION

DIS off the nucleon is parameterized by the hadronic tensor $W^{\mu\nu}(q)$ with q being the momentum transmitted to the nucleon. $W^{\mu\nu}(q)$ is obtained from the nucleon matrix element of the commutator $[J^\mu(\xi), J^\nu(0)]$. In the NJL model the current is conveniently given as $J^\mu = \bar{q}\mathcal{Q}\gamma^\mu q$, with \mathcal{Q} being the quark charge matrix. In the context of functional bosonization it is more appropriate to start from the forward virtual Compton amplitude,

$$T^{\mu\nu}(q) = \int d^4x e^{iq\cdot\xi} \langle N | T (J^\mu(\xi) J^\nu(0)) | N \rangle, \quad (4)$$

since the time-ordered product is unambiguously extracted from the regularized action

$$T (J^\mu(\xi) J^\nu(0)) = \frac{\delta^2}{\delta a_\mu(\xi) \delta a_\nu(0)} \text{Tr}_\Lambda \log [i\cancel{\partial} - (S + i\gamma_5 P) + \mathcal{Q}\cancel{\phi}] \Big|_{a_\mu=0}. \quad (5)$$

In this way the regularization of the structure functions is consistently implemented at the level of the defining action. The hadronic tensor is then obtained from the cut,

$$W^{\mu\nu}(q) = \frac{1}{2\pi} \Im (T^{\mu\nu}(q)). \quad (6)$$

In order to extract the leading twist piece of the structure functions, $W^{\mu\nu}(q)$ is studied in the Bjorken limit: $q^2 \rightarrow -\infty$ with $x = -q^2/P \cdot q$ fixed. Here P denotes the nucleon momentum. In this limit the leading order contribution in $1/N_C$ to $W^{\mu\nu}(q)$ becomes [3]

$$\begin{aligned} W^{\mu\nu}(q) &= i \frac{N_C}{4} \int \frac{d\omega}{2\pi} \sum_\alpha \int d^3\xi \int \frac{d\lambda}{2\pi} e^{iMx\lambda} \\ &\times \left\{ \left[\Psi_\alpha^\dagger(\vec{\xi}) \mathcal{Q}^2 \gamma^\mu \not{x} \gamma^\nu \beta \Psi_\alpha(\vec{\xi} + \lambda \hat{e}_3) e^{-i\lambda\omega} - \Psi_\alpha^\dagger(\vec{\xi}) \mathcal{Q}^2 \beta \gamma^\nu \not{x} \gamma^\mu \Psi_\alpha(\vec{\xi} + \lambda \hat{e}_3) e^{i\lambda\omega} \right] f_\alpha^{(-)}(\omega)_{\text{pole}} \right. \\ &\left. + \left[\Psi_\alpha^\dagger(\vec{\xi}) \mathcal{Q}^2 \beta \gamma^\mu \not{x} \gamma^\nu \Psi_\alpha(\vec{\xi} + \lambda \hat{e}_3) e^{-i\lambda\omega} - \Psi_\alpha^\dagger(\vec{\xi}) \mathcal{Q}^2 \gamma^\nu \not{x} \gamma^\mu \beta \Psi_\alpha(\vec{\xi} + \lambda \hat{e}_3) e^{i\lambda\omega} \right] f_\alpha^{(+)}(\omega)_{\text{pole}} \right\}, \quad (7) \end{aligned}$$

with the Pauli-Villars regularized spectral functions,

$$f_\alpha^{(\pm)}(\omega) = \sum_i c_i \frac{\omega \pm \epsilon_\alpha}{-\omega^2 + \epsilon_\alpha^2 + \Lambda_i^2 - i\eta} \pm \frac{\omega \pm \epsilon_\alpha}{-\omega^2 + \epsilon_\alpha^2 - i\eta}, \quad (8)$$

and $n^\mu = (1, 0, 0, 1)$ being a light-cone vector. As there are poles for both positive and negative ω , the meaning of forward and backward moving quarks becomes ambiguous and a description of $W_{\mu\nu}$ in terms of quark distributions seems impossible.

3. NUMERICAL RESULTS

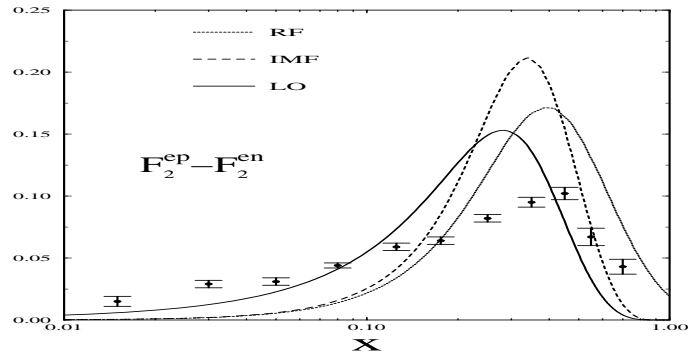
Unfortunately no numerical results are currently available for the structure functions as projected from the consistently regularized hadronic tensor (7). Therefore the presentation is limited to the valence quark approximation taking into account only the

contribution of the distinct valence level. Two observations make this a reliable approximation: (1) This level does not undergo regularization and hence the subtleties mentioned above are avoided. (2) Although the polarized vacuum is mandatory to provide a soliton solution, its contribution to static properties is small or negligible [2]. Sum rules relate them to structure functions, thus it is suggestive that the structure functions are also saturated by the valence quark contribution. For more details the reader is referred to the research papers [4, 5, 6] and similar studies by other groups [7, 8].

3.1. Unpolarized Structure Functions

The unpolarized structure functions are obtained from the symmetric combination $W_{\mu\nu} + W_{\nu\mu}$. The result for the structure function which enters the Gottfried sum rule of $e - N$ scattering is shown in fig. 1. The boost to the infinite momentum frame [9] mitigates the effects of omitting the dynamical response of the soliton to the infinite momentum transfer and provides proper support for the structure functions. A DGLAP evolution [10] determines the low energy scale $Q_0^2 \approx 0.4\text{GeV}^2$ at which the model supposedly approximates QCD. The model reproduces the gross features of the experimental data although an even better agreement can be gained by further reducing Q_0^2 which, however, would make the DGLAP program unrealistic. Depending on the model parameter m , the Gottfried sum rule, $S_G = \int dx(F_2^{\text{ep}} - F_2^{\text{en}})/x$ is found to be 0.26–0.29 which exhibits the desired deviation from the historic value (1/3) demanded empirically, 0.235 ± 0.026 [11].

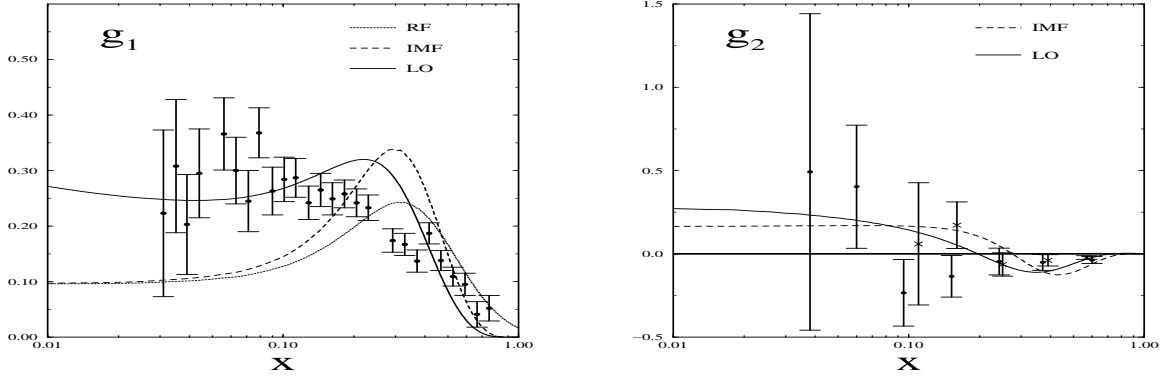
Fig. 1: The unpolarized structure function entering the Gottfried sum rule. RF: rest frame, IMF: boosted to the infinite momentum frame, LO: leading order QCD evolution to $Q^2 = 4\text{GeV}^2$. Data are from the NMC [11].



3.2. Spin Structure Functions

The spin – polarized structure functions g_1 and g_2 are obtained from the anti-symmetric combination $W_{\mu\nu} - W_{\nu\mu}$. While the former is related to the proton spin puzzle, the latter sheds some light on higher twist effects [12]. Typical results are shown in fig. 2 and are compared to data from SLAC [13]. Apparently a reasonable agreement is obtained.

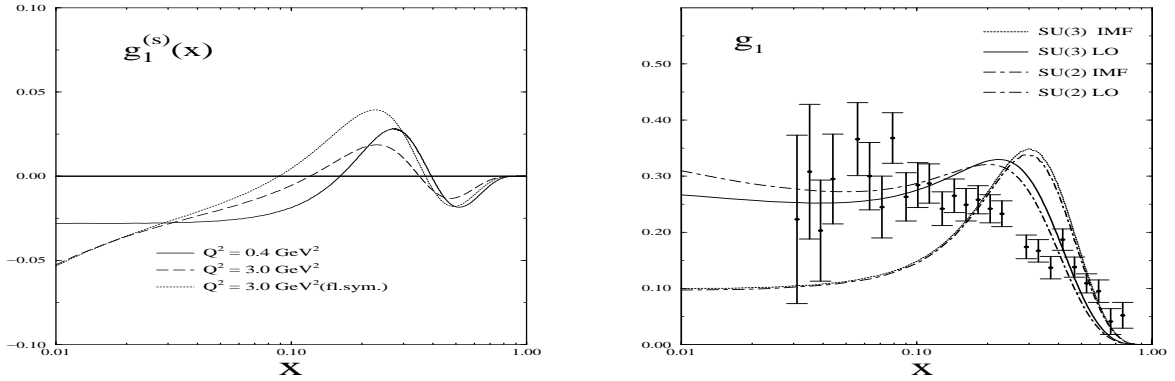
Fig. 2: Model predictions for the polarized nucleon structure functions g_1 (left panel) and g_2 (right panel). Data are from SLAC [13].



3.3. Generalization to Flavor SU(3)

The model can straightforwardly be generalized to also include strange quarks through the collective coordinate quantization [6]. In a flavor symmetric formulation one would expect sizable strange quark contributions to all nucleon properties. Fortunately flavor symmetry breaking effects can be included, thereby considerably reducing the strange quark contribution to g_1 , *cf.* fig. 3. This is also reflected by the small difference between the two and three flavor model calculation for g_1 of the proton.

Fig. 3: Left panel: Strangeness contribution to g_1 . Right panel: Comparison of the two and three flavor model calculation of g_1 of the proton. Data are from ref [13].



4. CONCLUSIONS

Nucleon structure functions can be computed from a chiral soliton model which describes baryons as lumps of mesons. All relevant information is contained in the hadronic tensor which can be computed from the symmetry currents of the model. Thus the identification of degrees of freedom with those of QCD, which seems impossible after regularization, is unnecessary. Reasonable agreement with experimental data is obtained in the valence quark approximation which leaves aside the technical subtleties of regularization.

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